## FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES DEPARTMENT OF MATHEMATICS AND STATISTICS

| QUALIFICATION: Bachelor of Science; Bachelor of Science in Applied Mathematics and Statistics |  |  |  |
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| QUALIFICATION CODE: | 07BOSC; 07BSAM | LEVEL: | 5 |
| COURSE CODE: | CLS502S | COURSE CODE: | CALCULUS 1 |
| SESSION: | NOVEMBER 2022 | PAPER: | THEORY |
| DURATION: | 3 HOURS | MARKS: | 100 |


| FIRST OPPORTUNITY EXAMINATION QUESTION PAPER |  |
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| EXAMINER: | DR. DSI IIYAMBO |
| MODERATOR: | DR. N CHERE |

## INSTRUCTIONS

1. Attempt all the questions in the booklet provided.
2. Show clearly all the steps used in the calculations
3. All written work must be done in black or blue inked, and sketches must be done in pencil.

## PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

## Question 1.

The functions $f, g$ and $h$ are defined by, $f(x)=\frac{2 x+1}{\sqrt{x^{2}+5 x+4}}, g(x)=x^{2}+3$ and $h(x)=2 x+a$.
a) Find the domain of $f$.
b) Given that $(g \circ h)(x)=4 x^{2}-8 x+7$, where $x \neq 0$, calculate the value of $a$.

## Question 2.

2.1 Find the following limits, if they exist.
a) $\lim _{h \rightarrow 0} \frac{\sqrt{4+h}-2}{h}$.
b) $\lim _{x \rightarrow 2^{-}} \frac{x^{2}-4}{|x-2|}$
c) $\lim _{x \rightarrow 0^{+}}\left(e^{x}+x\right)^{\frac{1}{x}}$
d) $\lim _{x \rightarrow 3} \frac{1}{(3-x)^{2}}$.
2.2 Using the Precise definition (the $\varepsilon-\delta$ method), prove that $\lim _{x \rightarrow-3}(14-5 x)=29$.

## Question 3.

a) Use the definition (first principle) to find the derivative of $f(x)=\sqrt{x+1}$.
b) Find the equation of the tangent line to the graph of $f$ at the point where $x=3$.
c) Find $g^{\prime}(x)$ for each of the following functions.
(i) $g(x)=\cos ^{2}(\cos x)$
(ii) $g(x)=3^{x} e^{x}$

## Question 4.

Consider the function $f(x)= \begin{cases}x-m & \text { if } x<3 ; \\ 1-m x & \text { if } x \geq 3 .\end{cases}$
a) Find the value of $m$ for which $f$ is a continuous function at $x=3$.
b) With the value of $m$ you found in a), is $f$ differentiable at $x=3$ or not? Justify your answer.

## Question 5.

Let $f(x)=x^{\frac{1}{3}}(2 x+7)$ and $g(x)=2 x-3 x^{\frac{2}{3}}$.
a) Find the intervals on which $f$ is increasing and on which it is decreasing, and hence state the local extreme values of $f$. If you answer is not a whole number, round it correct to 2 decimal places.
b) Find the intervals on which the graph of $y=g(x)$ is concave upwards and on which it is concave downwards.

